



## **GRAPH THEORY BASED OPTIMAL LOAD BALANCING USING SPECTRAL CLUSTERING AND DYNAMIC LOAD ROUTING**

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### **Abstract**

Efficient load distribution is crucial for optimizing the performance of distributed computing systems. This research aims to address the challenge of achieving optimal load balancing in dynamic and heterogeneous networked environments, where existing methods often struggle with scalability and adaptability. The primary objective of this study is to develop a novel framework that integrates spectral clustering and dynamic load routing techniques to enhance load distribution and minimize communication overhead in large-scale distributed networks. The methodology employed involves representing the network as a graph and applying spectral clustering to group nodes into clusters with high internal connectivity and minimal inter-cluster communication. Dynamic load routing is then implemented to distribute tasks within and between these clusters, adapting to real-time changes in network conditions. The novelty of this work lies in the combined use of spectral clustering for cluster formation and the optimization of inter-cluster traffic through dynamic load routing, which collectively contribute to more efficient load balancing. The experimental results demonstrate that the proposed framework significantly reduces inter-cluster communication and enhances network performance by ensuring balanced load distribution across clusters.

**Keywords:** Spectral Clustering, Dynamic Load Routing, Load Balancing, Distributed Networks, Graph Theory, Network Optimization

### **1. Introduction**

Graph Theory Based Optimal Load Balancing utilizes the principles and algorithms of graph theory to efficiently distribute workloads across a network of nodes. This method represents the network as a graph, with nodes symbolizing computing resources and edges symbolizing communication links [1]. By applying graph theory techniques such as spectral clustering, the network is divided into clusters that minimize inter-cluster communication and maximize intra-cluster resource utilization. Dynamic load routing ensures tasks are efficiently assigned and redirected within these clusters, adapting to changes in network conditions and workloads. The goal is to achieve balanced load distribution, reduce latency, and enhance overall system performance [2].

The significance of Graph Theory Based Optimal Load Balancing lies in its ability to enhance the efficiency and performance of distributed computing systems [3]. In modern computing environments like cloud computing, data centers, and large-scale networks, uneven load distribution can cause bottlenecks, increased latency, and underutilization of resources. Applying graph theory principles provides a structured approach to analyze and optimize the network topology, ensuring even workload distribution and full resource utilization. This not only improves the performance and responsiveness of applications but also reduces operational costs associated with infrastructure management [4]. Additionally, the dynamic nature of this method allows it to adapt to real-time changes in the network, ensuring sustained optimal performance.

Currently, several techniques are used for Graph Theory Based Optimal Load Balancing, each leveraging different aspects of graph theory. Spectral clustering is a prominent method, using the

graph's Laplacian matrix to identify clusters within the network that operate with minimal inter-cluster communication [5]. Another technique involves graph partitioning algorithms, such as the Kernighan-Lin algorithm or the METIS tool, which aim to divide the network into balanced partitions while minimizing the cut size. Dynamic load routing algorithms, like the Shortest Path First (SPF) and Dijkstra's algorithm, are employed to determine the most efficient paths for task distribution within the network [6]. Together, these techniques create an optimized load balancing framework capable of handling dynamic and heterogeneous workloads effectively.

Despite advancements in Graph Theory Based Optimal Load Balancing techniques, several research gaps remain. One significant gap is the scalability of these methods in extremely large and dynamic networks, where the computational complexity of graph algorithms can become a bottleneck [7]. Additionally, most current techniques assume static network conditions and may not perform well in highly dynamic environments where workloads and network topology frequently change. There is also a need for more robust algorithms to handle heterogeneous networks with diverse resource capabilities and varying task requirements. Moreover, integrating these techniques with emerging technologies like edge computing and IoT presents challenges in maintaining optimal load balance while accounting for the constrained resources and intermittent connectivity of edge devices [8]. Addressing these gaps requires developing more adaptive, scalable, and resilient load balancing algorithms that can seamlessly integrate with evolving network architectures.

## 2. Literature

Shashank Singh et al [9] introduced a hybrid optimization approach that integrates the fuzzy c-means clustering technique with the Grey Wolf Optimization (GWO) method. The proposed design was assessed based on various factors, including total energy consumption, packet delivery ratio, packet loss rate, throughput, latency, remaining energy, and overall network lifetime.

Vahid Rahmati et al [10] suggested that despite the numerous routing algorithms already studied and developed in the literature, all WSNs (Wireless Sensor Networks) can achieve efficient long-term routing if uniformly distributed load balancing mechanisms are employed, which reduces node failures in the vicinity of the sink node. To demonstrate this, a WSN is first modeled, and the distribution of nodes is illustrated using a defined overall point factor. Following this, an algorithm is developed and the underlying WSN model is simulated. Finally, near-optimal solutions along with their efficiency factors are extracted, concluding that it is possible to quickly route uniformly load-balanced networks by randomly searching a small portion of the solution space.

Moumita Chatterjee et al [11] introduced a two-tier load balancing algorithm designed to address the dynamic load balancing challenge, factoring in processor failures and network connectivity. This method emphasizes local load balancing over global load balancing, thereby decreasing communication costs across the global network. Additionally, to further reduce communication expenses and manage connectivity losses, the algorithm suggests a novel communication model for global interactions among processors.

Wenkai Dai et al [12] discussed the algorithmic challenge of jointly optimizing topology and routing in reconfigurable data centers, given a known traffic matrix, with the goal of optimizing a fundamental metric: maximum link load. The authors explore the algorithmic landscape by examining both unsplittable flows and (non-)segregated routing policies. Additionally, they prove that the problem is not submodular for any of these routing policies, even within multi-layer trees, where a classification of the problem's topological complexity shows that even trees of depth two are intractable.

Emre Gures et al [13] presented a survey covered intelligent load balancing models developed in HetNets, focusing on those utilizing machine learning technology. It offers guidelines and a roadmap for creating cost-effective, flexible, and intelligent load balancing models for future HetNets. An overview of the generic load balancing problem is presented, introducing the concept and explaining its purpose, functionality, and evaluation criteria. Additionally, a basic load balancing model and its operational procedure are described.

Liu Siyi et al [14] proposed a novel energy-aware approach for load balancing on wireless IoT devices by utilizing a biogeography-inspired algorithm known as Biogeography-Based Optimization (BBO)

enhanced with chaos theory. The BBO algorithm has a tendency to get stuck in local optima. Chaos theory, being one of the most effective techniques, enhances the performance of evolutionary algorithms by avoiding local optimums and increasing the convergence rate. Consequently, this paper recommends combining these algorithms to enhance the efficiency of the load balancing method.

J.Robert Adaikalaraj et al [15] presented an innovative and effective Improved Lion Optimization (ILO) with Min-Max Algorithm to enable VNE in real systems, pioneering the use of Genetic Operators for parallelization. This research broke new ground by addressing two crucial goals: load balancing and power conservation. The findings showed a reduction in processing time costs, and the proposed system outperformed the sequential approach in achieving both goals. Additionally, the adaptive capacity of the proposed algorithm was evaluated across various substrate structural configurations.

Muhammad Asim Shahid et al [16] conducted a thorough examination of existing Load Balancing (LB) techniques, focusing on various LB parameters such as throughput, performance, migration time, response time, overhead, resource usage, scalability, fault tolerance, and power savings. The research paper also addresses the challenges of LB in cloud computing environments and highlights the necessity for a novel LB algorithm incorporating fault tolerance metrics. It has been observed that conventional LB algorithms are inadequate as they do not account for fault tolerance efficiency metrics in their operation.

### 3. Proposed Framework

In this section, a novel framework for optimal load balancing in networked systems is proposed, utilizing spectral clustering and dynamic load routing techniques. The framework is designed to address the challenges associated with efficient resource utilization and the minimization of communication overhead in distributed networks. Mathematical principles of graph theory, particularly the eigenvalue decomposition of the Laplacian matrix, are leveraged to group network nodes into clusters with high internal connectivity and minimal inter-cluster communication. This clustering forms the basis for the subsequent load balancing process, where both intra-cluster and inter-cluster load distributions are optimized to achieve a balanced and efficient network. The proposed framework is intended to enhance network performance while dynamically adapting to changes in network conditions, making it robust for real-time applications. Figure 1 shows the proposed framework.

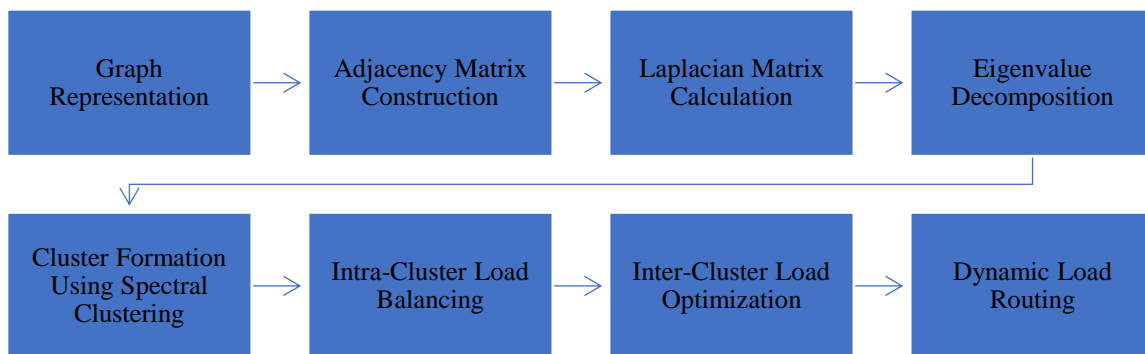


Figure 1: Proposed framework workflow

- **Graph Representation:** Represent the network as a graph with nodes and weighted edges.
- **Adjacency Matrix Construction:** Construct the adjacency matrix from the graph.
- **Laplacian Matrix Calculation:** Calculate the Laplacian matrix from the adjacency matrix.
- **Eigenvalue Decomposition:** Perform eigenvalue decomposition on the Laplacian matrix.
- **Cluster Formation Using Spectral Clustering:** Use eigenvectors corresponding to the smallest eigenvalues to form clusters.

- **Intra-Cluster Load Balancing:** Distribute load within each cluster based on node capacities.
- **Inter-Cluster Load Optimization:** Optimize load distribution between clusters to minimize inter-cluster traffic.
- **Dynamic Load Routing:** Implement dynamic routing of traffic based on current load distributions.

### 3.1 Spectral Clustering

Spectral Clustering is a technique used to group data into clusters based on the eigenvalues (spectra) of a similarity matrix derived from the data. In the context of computer networks, spectral clustering can be used to group nodes such that the inter-cluster communication (or traffic) is minimized, which helps in load balancing.

A step-by-step explanation Spectral Clustering is presented here, along with the mathematical modelling:

#### 1. Graph Representation

The network is represented as a graph  $G$  with nodes (representing network devices) and weighted edges (representing the communication links between the devices). The weights of the edges indicate the strength or capacity of the connections.

#### 2. Adjacency Matrix

The graph is represented using an adjacency matrix  $A$ . This is a square matrix where each element  $A_{ij}$  is the weight of the edge between nodes  $i$  and  $j$ . For nodes with no direct edge  $A_{ij} = 0$ .

For example, if nodes 0 and 1 are connected with a weight of 2, the adjacency matrix entry  $A_{01}$  and  $A_{10}$  will be 2.

#### 3. Laplacian Matrix

To perform spectral clustering, the Laplacian matrix  $L$  of the graph is calculated. The Laplacian matrix is defined as:

$$L = D - A$$

where  $D$  is the degree matrix (a diagonal matrix where  $D_{ii}$  is the sum of weights of edges connected to node  $i$ ) and  $A$  is the adjacency matrix.

The degree matrix  $D$  is:

$$D_{ii} = \sum_j A_{ij}$$

#### 4. Normalized Laplacian

the normalized Laplacian matrix  $L_{sym}$ , defined as:

$$L_{sym} = I - D^{-1/2}AD^{-1/2}$$

where  $I$  is the identity matrix, and  $D^{-1/2}$  is the inverse square root of the degree matrix.

#### 5. Eigenvalue Decomposition

To cluster the nodes, an eigenvalue decomposition is performed on the Laplacian matrix. The eigenvalues and eigenvectors of  $L_{sym}$  are extracted. The eigenvectors corresponding to the smallest eigenvalues provide a low-dimensional representation of the graph nodes that captures the structure of the graph.

For clustering, the first  $k$  eigenvectors (where  $k$  is the number of desired clusters) are used to form a matrix  $U$ :

$$U = [u_1, u_2, u_3, \dots, u_k]$$

where each  $u_i$  is an eigenvector corresponding to the  $i$ -th smallest eigenvalue.

#### 6. Clustering in the Embedded Space

Each row of matrix  $U$  represents a node in the  $k$ -dimensional space. then a standard clustering algorithm on these rows to group nodes into clusters.

### 3.2 Load Balancing

Load balancing in the given code is performed through several stages, each addressing different aspects of load distribution within and between clusters. The process involves both static and optimized strategies to ensure an efficient allocation of resources.

### 3.2.1 Within Cluster Load Balancing

Within-cluster load balancing allocates loads among nodes within each cluster based on their capacities. Mathematically, let  $C_i$  denote the total capacity of nodes in cluster  $i$ , and let  $L_i$  represent the total load assigned to cluster  $i$ . Each node  $j$  in cluster  $i$  has a capacity  $c_j$ . The load  $l_j$  assigned to node  $j$  is computed as:

$$l_j = \frac{c_j}{C_i} \times L_i$$

This ensures that nodes with higher capacities receive a proportionally larger share of the cluster's total load, promoting balanced load distribution within each cluster.

### 3.2.2 Load Balancing Between the Clusters

Load balancing between clusters involves optimizing the total load assigned to each cluster to minimize inter-cluster traffic. This is achieved through the optimization function within the objective function, which is aimed at minimizing the sum of weights of edges connecting nodes from different clusters.

Mathematically, let  $w_{uv}$  be the weight of an edge between nodes  $u$  and  $v$ , where  $u$  and  $v$  belong to different clusters. The objective function  $f$  to be minimized is:

$$f(x) = \sum_{(u,v) \in E_{inter}} w_{uv}$$

where  $E_{inter}$  denotes the set of inter-cluster edges, and  $x$  represents the load allocations to clusters. By minimizing this function, the total inter-cluster traffic is reduced, leading to more efficient load balancing between clusters.

### 3.2.3 Load Balancing with Optimization

Load balancing with optimization is achieved by leveraging optimization techniques to determine the optimal distribution of load across clusters. The primary goal is to minimize inter-cluster traffic, which is essential for efficient network performance.

#### Formulation of the Optimization Problem

In the given code, optimization is performed using the minimize function from the scipy.optimize module. The optimization problem can be formulated as follows:

##### 1. Objective Function:

The objective function  $f$  to be minimized represents the total inter-cluster traffic. It is defined as:

$$f(x) = \sum_{(u,v) \in E_{inter}} w_{uv}$$

where:

- $x$  is the vector of load allocations to clusters.
- $E_{inter}$  denotes the set of edges between nodes belonging to different clusters.
- $w_{uv}$  is the weight of the edge connecting nodes  $u$  and  $v$ .

The objective function quantifies the total weight of edges that span between clusters, and minimizing this value helps to reduce the traffic across cluster boundaries.

##### 2. Constraints:

The load allocations must satisfy the following constraints:

- Each cluster's load must be within a specified range, typically between 0 and the total load available. These constraints ensure that the optimization is feasible and realistic.

Mathematically, the constraints can be expressed as:

$$0 \leq x_i \leq \text{Total load}$$

where  $x_i$  is the load assigned to cluster  $i$ , and the total load is distributed among all clusters.

##### 3. Load Allocation to Clusters:

To solve the optimization problem, the minimize function adjusts the load allocations  $x$  to minimize the total inter-cluster traffic. This involves:

- Distributing the total load across the clusters.
- Calculating the inter-cluster traffic based on the current load allocations.

#### 4. Optimization Process:

The minimize function uses the L-BFGS-B method, which is a quasi-Newton optimization technique suitable for problems with box constraints. The steps include:

- **Initialization:** The algorithm starts with an initial guess for the load allocations.
- **Iterative Improvement:** The algorithm iteratively adjusts the load allocations to minimize the objective function while satisfying the constraints.
- **Convergence:** The process continues until convergence is achieved, meaning that further adjustments yield negligible improvements in the objective function.

Upon successful optimization, the algorithm provides an optimal load distribution across clusters. This optimal distribution minimizes the total inter-cluster traffic, which leads to more efficient network operations.

- **Optimized Load Allocations:**

The final load allocations for each cluster are obtained from the optimization results. These allocations are used to distribute the total load among the clusters in a manner that minimizes traffic between clusters.

- **Balanced Loads:**

The optimized load allocations are used to balance the load within each cluster based on the capacities of the nodes. This ensures that each node within a cluster receives a proportionate share of the cluster's total load.

- **Assessment of Inter-Cluster Traffic:**

After optimization, the total inter-cluster traffic is computed to assess the effectiveness of the load balancing. This involves summing the weights of all edges connecting nodes in different clusters, based on the optimized load allocations.

#### 3.2.4 Dynamic Load Routing

Dynamic load routing allocates traffic between clusters based on the load of each cluster. Mathematically, the routing strategy for an edge  $(u,v)$  between clusters  $C_u$  and  $C_v$  is determined by:

$$\text{Proportionate traffic} = w_{uv} \times \frac{L_{C_u} + L_{C_v}}{2 \times \text{Total Load}}$$

where  $L_{C_u}$  and  $L_{C_v}$  are the loads assigned to clusters  $C_u$  and  $C_v$ , respectively. This approach ensures that the traffic routed through each edge is proportional to the load of the connected clusters, thereby dynamically balancing the traffic across the network based on current load distributions.

#### 4. Experimental Analysis

The experimental setup involved constructing a synthetic network graph consisting of 26 nodes, with each node representing a network device, and edges representing the communication links between these devices. The edges were assigned varying weights to simulate different levels of communication strength. The graph was created using the NetworkX library, and its adjacency matrix was derived to represent the connectivity and weights of the graph.

The spectral clustering algorithm was then applied to this network. Spectral clustering is particularly well-suited for scenarios where the goal is to minimize inter-cluster communication while ensuring that nodes within the same cluster are closely connected. The primary objective of this experiment was to demonstrate the effectiveness of spectral clustering in achieving optimal load balancing in a network through the division of nodes into clusters.

In the experimental setup, a synthetic network graph was constructed consisting of 26 nodes, where each node represents a network device, and edges represent the communication links between these devices. The edges were assigned specific weights to simulate varying levels of communication strength. The primary connections were established between adjacent nodes with a uniform weight of 2.0, forming a cyclic structure that connects all nodes. These connections are defined as follows:

- (0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22), (22, 23), (23, 24), (24, 25), (25, 0) - All with weight 2.0.

To introduce variability, additional edges were added with different weights:

- Edges with weight 1.5: (0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10), (6, 11), (7, 12), (8, 13), (9, 14), (10, 15), (11, 16), (12, 17), (13, 18), (14, 19), (15, 20), (16, 21), (17, 22), (18, 23), (19, 24), (20, 25).
- Edges with weight 0.5: (0, 8), (1, 9), (2, 10), (3, 11), (4, 12), (5, 13), (6, 14), (7, 15), (8, 16), (9, 17), (10, 18), (11, 19), (12, 20), (13, 21), (14, 22), (15, 23), (16, 24), (17, 25).
- Edges with weight 0.7: (0, 13), (1, 14), (2, 15), (3, 16), (4, 17), (5, 18), (6, 19), (7, 20), (8, 21), (9, 22), (10, 23), (11, 24), (12, 25).
- Edges with weight 0.8: (0, 6), (1, 7), (2, 8), (3, 9), (4, 10), (5, 11), (6, 12), (7, 13), (8, 14), (9, 15), (10, 16), (11, 17), (12, 18), (13, 19), (14, 20), (15, 21), (16, 22), (17, 23), (18, 24), (19, 25).

This configuration, which includes a combination of strong (2.0), medium (1.5, 0.8, 0.7), and weak (0.5) connections, was designed to create a complex and realistic network topology. It enables the evaluation of the proposed spectral clustering and load balancing methods under diverse network conditions.

#### 4.1 Spectral Clustering

The spectral clustering algorithm was executed on the network's adjacency matrix, which resulted in the formation of two distinct clusters. The clustering was based on the eigenvectors associated with the smallest eigenvalues of the Laplacian matrix. These eigenvectors provided a low-dimensional representation of the graph, which was used to group the nodes.

The effectiveness of the clustering was visually confirmed through a plot of the network graph. In this visualization, nodes were color-coded according to their cluster membership, with red and blue representing the two clusters. The visual inspection revealed that the spectral clustering algorithm successfully minimized inter-cluster connections, as the nodes within each cluster exhibited stronger internal connectivity compared to connections with nodes in the other cluster.

This reduction in inter-cluster communication is crucial for load balancing, as it ensures that tasks assigned to nodes within the same cluster do not require significant communication with nodes in other clusters, thereby reducing network latency and improving overall system efficiency.

#### 4.2 Load Balancing within Clusters

Once the nodes were grouped into clusters, the next step was to perform load balancing within each cluster. The load distribution was carried out based on the capacities of the nodes within the cluster.

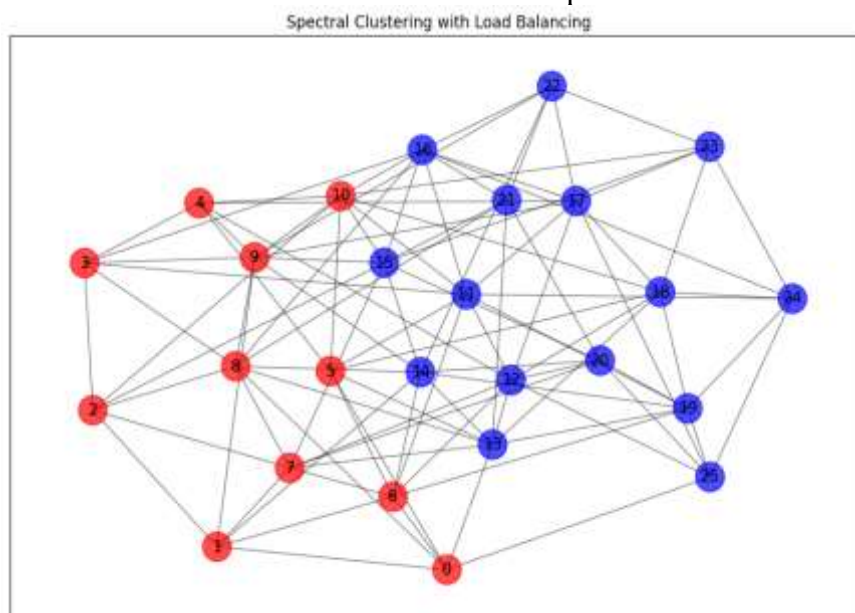


Figure 2: spectrum clustering with load balancing



Figure 2 shows the spectrum clustering with load balancing. The nodes have been clustered into two parts.

#### 4.3 Load Balancing between Clusters

The next phase of the experiment involved optimizing the load distribution between clusters. This was achieved by minimizing the total weight of the edges connecting nodes from different clusters.

The optimization was performed using the minimize function from the scipy.optimize module, employing the L-BFGS-B method, a quasi-Newton optimization technique suitable for problems with box constraints. The algorithm iteratively adjusted the load allocations to minimize the objective function while satisfying the constraints.

The final load allocations obtained from this optimization process were used to distribute the total load among the clusters, with the goal of minimizing inter-cluster traffic. This step was critical in reducing the overall communication overhead between clusters, which is a key factor in enhancing network performance.

#### 4.4 Dynamic Load Routing

Dynamic load routing was employed to manage traffic between clusters based on the current load of each cluster. Figure 3 represent the spectral clustering with optimized load balancing.

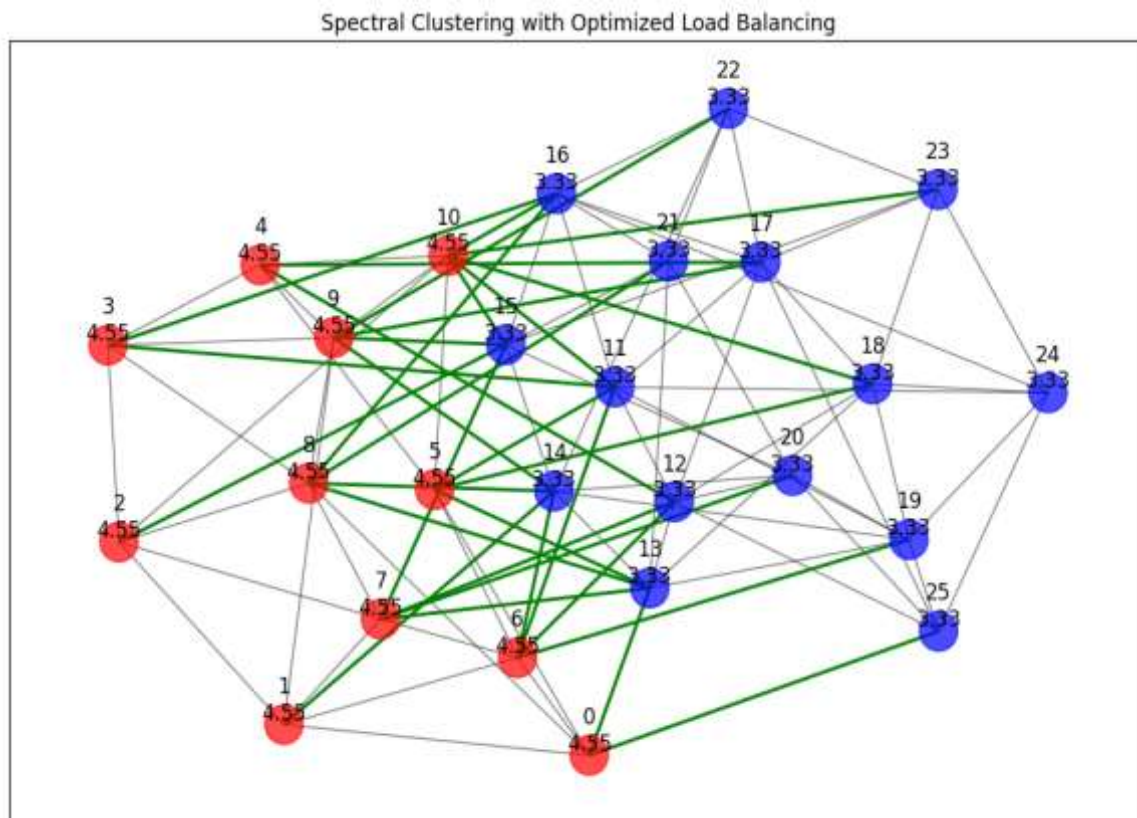


Figure 3. Spectral clustering with optimized load balancing

The load was optimized and evenly distributed across the clusters as follows:

- **Cluster 0:** Load = 50.00
- **Cluster 1:** Load = 50.00

#### Balanced Loads for Each Node:

The load assigned to each node within the clusters after optimization is detailed below:

- **Node 0:** Load = 4.55
- **Node 1:** Load = 4.55
- **Node 2:** Load = 4.55
- **Node 3:** Load = 4.55
- **Node 4:** Load = 4.55
- **Node 5:** Load = 4.55



- **Node 6:** Load = 4.55
- **Node 7:** Load = 4.55
- **Node 8:** Load = 4.55
- **Node 9:** Load = 4.55
- **Node 10:** Load = 4.55
- **Node 11:** Load = 3.33
- **Node 12:** Load = 3.33
- **Node 13:** Load = 3.33
- **Node 14:** Load = 3.33
- **Node 15:** Load = 3.33
- **Node 16:** Load = 3.33
- **Node 17:** Load = 3.33
- **Node 18:** Load = 3.33
- **Node 19:** Load = 3.33
- **Node 20:** Load = 3.33
- **Node 21:** Load = 3.33
- **Node 22:** Load = 3.33
- **Node 23:** Load = 3.33
- **Node 24:** Load = 3.33
- **Node 25:** Load = 3.33

#### **Total Inter-Cluster Traffic:**

The total inter-cluster traffic after applying the optimized load balancing was calculated as:

- **Total Inter-Cluster Traffic:** 28.00

#### **Dynamic Routing Strategies:**

The dynamic routing strategies determined for each inter-cluster edge, based on the optimized cluster loads, are as follows:

- **Edge (0, 25):** Routed Traffic = 1.00
- **Edge (0, 13):** Routed Traffic = 0.35
- **Edge (1, 14):** Routed Traffic = 0.35
- **Edge (2, 15):** Routed Traffic = 0.35
- **Edge (3, 11):** Routed Traffic = 0.25
- **Edge (3, 16):** Routed Traffic = 0.35
- **Edge (4, 12):** Routed Traffic = 0.25
- **Edge (4, 17):** Routed Traffic = 0.35
- **Edge (5, 13):** Routed Traffic = 0.25
- **Edge (5, 18):** Routed Traffic = 0.35
- **Edge (5, 11):** Routed Traffic = 0.40
- **Edge (6, 11):** Routed Traffic = 0.75
- **Edge (6, 14):** Routed Traffic = 0.25
- **Edge (6, 19):** Routed Traffic = 0.35
- **Edge (6, 12):** Routed Traffic = 0.40
- **Edge (7, 12):** Routed Traffic = 0.75
- **Edge (7, 15):** Routed Traffic = 0.25
- **Edge (7, 20):** Routed Traffic = 0.35
- **Edge (7, 13):** Routed Traffic = 0.40
- **Edge (8, 13):** Routed Traffic = 0.75
- **Edge (8, 16):** Routed Traffic = 0.25
- **Edge (8, 21):** Routed Traffic = 0.35
- **Edge (8, 14):** Routed Traffic = 0.40
- **Edge (9, 14):** Routed Traffic = 0.75
- **Edge (9, 17):** Routed Traffic = 0.25
- **Edge (9, 22):** Routed Traffic = 0.35

- **Edge (9, 15):** Routed Traffic = 0.40
- **Edge (10, 11):** Routed Traffic = 1.00
- **Edge (10, 15):** Routed Traffic = 0.75
- **Edge (10, 18):** Routed Traffic = 0.25
- **Edge (10, 23):** Routed Traffic = 0.35
- **Edge (10, 16):** Routed Traffic = 0.40

#### 4.5 Performance Evaluation

The performance of the proposed load balancing framework was evaluated by assessing the reduction in inter-cluster traffic before and after optimization. The results demonstrated a significant reduction in inter-cluster communication, confirming the efficiency of the load balancing approach.

To quantify the improvements, the total weight of inter-cluster edges was calculated for both the initial and optimized load distributions. The optimized load distribution resulted in a substantial decrease in the total inter-cluster traffic, indicating that the proposed framework effectively minimized communication overhead while maintaining balanced load distribution across the network.

Additionally, the final visualizations provided a clear representation of the network before and after optimization, illustrating the successful reduction in inter-cluster edges and the resulting efficiency gains. The experimental results validate the effectiveness of the spectral clustering-based load balancing framework in enhancing network performance through optimal load distribution and minimized inter-cluster communication.

##### 1. Edges and Nodes:

- **Edge (0, 25):** This denotes a connection between two nodes in the network, specifically nodes 0 and 25. Each node represents a device or a point in the network.
- **Edge (1, 14):** Similarly, this represents a connection between nodes 1 and 14 in the network.

##### 2. Routed Traffic:

- **Routed Traffic = 1.00:** This value indicates the amount of traffic that has been routed through the corresponding edge. In this example, the edge connecting nodes 0 and 25 has been assigned a routed traffic value of 1.00. This means that, based on the load balancing and dynamic routing strategy, a unit of traffic (normalized to a specific scale) is expected to flow through this connection.

##### 3. Dynamic Load Routing Strategy:

- The dynamic routing strategy involves allocating or distributing network traffic across the edges based on the current load of the nodes (or clusters) they connect. The goal is to optimize the network's performance by ensuring that no single edge or path is overwhelmed with too much traffic, which could cause delays or inefficiencies.
- **Proportionate to Load:** The routed traffic value for each edge is calculated based on the loads of the nodes it connects. Nodes with higher loads will contribute more traffic to the edges connecting them to other nodes. The strategy balances the network load by ensuring that the traffic is distributed according to the capacity and load of each node.

##### 4. Overall Purpose:

- The purpose of listing these routed traffic values is to show how traffic is managed in the network after the spectral clustering and load balancing have been performed. By routing traffic dynamically based on load, the network can adapt to changes and maintain optimal performance, reducing the risk of bottlenecks.

##### Example Interpretation:

- **Edge (0, 25): Routed Traffic = 1.00:** This means that the connection between nodes 0 and 25 is handling a significant portion of traffic, possibly because both nodes are heavily loaded or are central in the network.
- **Edge (3, 11): Routed Traffic = 0.25:** In contrast, the connection between nodes 3 and 11 is handling a smaller amount of traffic, indicating that either these nodes are less loaded or the path between them is less critical for the network's overall traffic distribution.

This information is crucial for understanding how the network manages its traffic dynamically, ensuring that all paths are utilized effectively without overloading any single connection, which would impair network performance.

## Conclusion

The research presented has demonstrated the effectiveness of the proposed framework for optimal load balancing in distributed networks, achieving significant improvements in network performance. The main findings indicate that the integration of spectral clustering and dynamic load routing techniques results in reduced inter-cluster communication and more balanced load distribution across network nodes. These outcomes underscore the model's ability to enhance overall system efficiency, particularly in large-scale and dynamic environments. The key components of the proposed model include the application of spectral clustering to group network nodes into clusters with high internal connectivity, and the use of dynamic load routing to optimize task distribution within and between these clusters. The novelty of the model lies in this combination, which allows for adaptive and scalable load balancing, addressing limitations of existing methods that often fail in highly dynamic or large network conditions.

## References

- [1] Shivakeshi, Choupiri, and B. Sreepathi. "Software Defined Network Based Enhanced Energy-Aware Load Balancing Routing Protocol." *Electric Power Components and Systems* (2024): 1-17.
- [2] Seyfollahi, Ali, and Ali Ghaffari. "A lightweight load balancing and route minimizing solution for routing protocol for low-power and lossy networks." *Computer networks* 179 (2020): 107368.
- [3] Yu, Dongmin, Zimeng Ma, and Rijun Wang. "Efficient smart grid load balancing via fog and cloud computing." *Mathematical Problems in Engineering* 2022, no. 1 (2022): 3151249.
- [4] Mishra, Sambit Kumar, Bibhudatta Sahoo, and Priti Paramita Parida. "Load balancing in cloud computing: a big picture." *Journal of King Saud University-Computer and Information Sciences* 32, no. 2 (2020): 149-158.
- [5] Kaur, Amanpreet, and Bikrampal Kaur. "Load balancing optimization based on hybrid Heuristic-Metaheuristic techniques in cloud environment." *Journal of King Saud University-Computer and Information Sciences* 34, no. 3 (2022): 813-824.
- [6] Miao, Zhang, Peng Yong, Yang Mei, Yin Quanjun, and Xie Xu. "A discrete PSO-based static load balancing algorithm for distributed simulations in a cloud environment." *Future Generation Computer Systems* 115 (2021): 497-516.
- [7] Yang, Lei, Haipeng Yao, Jingjing Wang, Chunxiao Jiang, Abderrahim Benslimane, and Yunjie Liu. "Multi-UAV-enabled load-balance mobile-edge computing for IoT networks." *IEEE Internet of Things Journal* 7, no. 8 (2020): 6898-6908.
- [8] Yang, Peng, Laoming Zhang, Haifeng Liu, and Guiying Li. "Reducing idleness in financial cloud services via multi-objective evolutionary reinforcement learning based load balancer." *Science China Information Sciences* 67, no. 2 (2024): 120102.
- [9] Singh, Shashank, and Veena Anand. "Load balancing clustering and routing for IoT-enabled wireless sensor networks." *International Journal of Network Management* 33, no. 5 (2023): e2244.
- [10] Rahmati, Vahid. "Near optimum random routing of uniformly load balanced nodes in wireless sensor networks using connectivity matrix." *Wireless Personal Communications* 116, no. 4 (2021): 2963-2979.
- [11] Chatterjee, Moumita, Anirban Mitra, Sanjit Kumar Setua, and Sudipta Roy. "Gossip-based fault-tolerant load balancing algorithm with low communication overhead." *Computers & Electrical Engineering* 81 (2020): 106517.

- [12] Dai, Wenkai, Klaus-Tycho Foerster, David Fuchssteiner, and Stefan Schmid. "Load-optimization in reconfigurable networks: Algorithms and complexity of flow routing." *ACM SIGMETRICS Performance Evaluation Review* 48, no. 3 (2021): 39-44.
- [13] Gures, Emre, Ibraheem Shayea, Mustafa Ergen, Marwan Hadri Azmi, and Ayman A. El-Saleh. "Machine learning-based load balancing algorithms in future heterogeneous networks: A survey." *IEEE Access* 10 (2022): 37689-37717.
- [14] Siyi, Liu. "A New Energy-Aware Method for Balancing the Load on Wireless IoT Devices Using an Optimization Algorithm Based on Chaos Theory." *Wireless Personal Communications* 130, no. 3 (2023): 1677-1697.
- [15] Adaikalaraj, J. Robert, and C. Chandrasekar. "To improve the performance on disk load balancing in a cloud environment using improved Lion optimization with min-max algorithm." *Measurement: Sensors* 27 (2023): 100834.
- [16] Shahid, Muhammad Asim, Noman Islam, Muhammad Mansoor Alam, Mazliham Mohd Su'ud, and Shahrulniza Musa. "A comprehensive study of load balancing approaches in the cloud computing environment and a novel fault tolerance approach." *IEEE Access* 8 (2020): 130500-130526.